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DEVELOPMENT OF A MATHEMATICAL MODEL FOR TEMPERATURE DISTRIBUTION IN A HEATING SYSTEM

This paper presents a mathematical model of heat transfer fluid flow distribution for creating a digital twin of a district heating system, using the district heating networks of Pavlodar as an example. Modeling such systems allows for the optimization of heat transfer fluid distribution, taking into account various parameters such as temperature, pressure, flow rate, and pipeline characteristics. As an example, a mathematical model of temperature distribution in a heating network has been developed based on the heat exchange process with the environment, and options for above-ground and underground (tunnel) piping have been considered. The description of temperature distribution along the length of the pipeline between network nodes has been utilized.

The aim of the study is to create an effective mathematical model that can be used to calculate and optimize heat transfer fluid flows in heating systems. The model can be applied to the design of new systems and the modernization of existing ones, as well as for real-time operational control of heat transfer fluid flows.

The main objectives of the study include analyzing existing methods and approaches to modeling heat transfer fluid flows, developing a mathematical model that accounts for the characteristics of heating networks, and verifying and validating the model based on experimental data.

The results of this research may be useful for specialists in the field of heat supply, design organizations, operating companies, and researchers working on issues of energy efficiency and optimization of heat supply systems.

Keywords: mathematical model, digital twin, heat supply, thermal calculation, hydraulic calculation.

Introduction

Implementation of digital twin into the work of housing and communal services is rentable in order to optimize the work of heat supply system, prediction the emergencies, heightening efficiency and reduction the expanses. The use of a backup system contributes the stability of energy supply for objects with minimal spends and the anticipations of emergency changes [1].

In order to implement the digital twin in the first place is necessary to obtain adequate mathematical model of real system, which will take into account various factor and choose the optimal solutions of assigned tasks. The point of view of analysis and failure prediction it is obligatory in base of previously received and processed data letting reduce the risk for consumers and minimization of downtime and cost of servicing heating system. The possession of digital model enables testing new solutions before implementation in a real system; this will reduce the risks and costs of experiments. For consumer the digital twin will provide precise data in the shortest possible time, making maintenance more transparent increasing trust and complacency of consumers. Modeling assists to ensure that the heating system corresponds all established requirements and standards [2].

Materials and methods

First step making the adequate model is collection and analysis of initial data. By defying the parameters of heating system, including the characteristics of pipeline, pumps, heat exchangers and other elements, temperature and pressure data and reliable operation, one can begin to select a suitable modeling method.

Let us consider, as an example, the development of a mathematical model for temperatures distribution in a heating network based on the process of heat exchange with the environment [3].

Temperature distribution along the section of pipeline (feed or return) between the nodes of heating network is described by following heat transfer equation:

$$u \frac{\partial T}{\partial x} + \frac{4K}{\rho DC_p} (T - T_{env}) = 0, \quad (1)$$

where x is the coordinate along the section of a pipeline;

T is the coolant temperature (water);

ρ, C_p are the density and specific heat capacity of water;

D is the internal diameter of pipeline;

u is the average flow velocity across the cross-section of a pipe expressed in terms of water consumption q : $u = \frac{q}{0.25\pi D^2}$;

K is the heat exchange coefficient from the coolant flow to the environment;

T_{env} is the temperature of the environment

The equation has the following solution, called Shukhov's formula:

$$T(x) = T_{env} + (T_0 - T_{env})e^{-\frac{\pi \rho K D x}{q C_p}} \quad (2)$$

The value of K is defined from following ratios:

$$\frac{1}{K\pi D} = R = R_{in} + R_{ins} + R_{out} \quad (3)$$

where R is the general thermal resistance of heat transfer pipe;

R_{in} is the thermal resistance heat transfer from the coolant to the pipe wall;

R_{ins} is the thermal resistance of the pipe insulation coating;

R_{out} is the thermal resistance from the outer surface of the pipe to the external environment.

The value R_{in} was defined by means of the Nusselt number Nu , expressed via Reynolds number Re and Prandtl number Pr [4]:

$$R_{in} = \frac{1}{\lambda_w \pi Nu}, \quad Nu = 0.021 \cdot (Re)^{0.8} \cdot (Pr)^{0.43} \quad (4)$$

$$Re = \frac{uD}{\nu}, \quad Pr = \frac{\nu \rho C_p}{\lambda_w}$$

where λ_w is the thermal conductivity of water;

ν is the kinematic viscosity of water.

The R_{ins} value was determined from the parameters of isolation coating:

$$R_{ins} = \frac{1}{2\pi\lambda_{ins}} \ln \frac{D_2}{D_1} \quad (5)$$

where D_1, D_2 are internal and external diameters of the pipeline insulation coating layer;

λ_{ins} is the thermal conductivity coefficient of the insulation layer.

The T_{env} value in formula (2) and the R_{out} value in ratio (3) were determined depending on the type of pipeline installation (overground, underground channelless and underground channel). As a rule, each sector of the heating network contains two pipes: supply and return pipes, which have various temperatures of the coolant. In contrast with underground, the overground insulation the influence of temperature field parallel to the pipe laid (adjacent) to the pipe in question was not considered [4].

Results and consideration

For above-ground pipeline installation, the outside air was considered as the ambient air, the value was calculated using the following formula [5]:

$$T_{env} = T_{air}, \quad R_{out} = \frac{1}{\alpha_{air}\pi D_2}, \quad \alpha_{air} = 11.6 + \sqrt{w}, \quad (6)$$

where α_{air} is the coefficient of heat transfer from the outer surface of the pipe to the surrounding air;

w is the wind velocity;

T_{air} is the outside air temperature.

For above-ground pipeline installation, the outside air was considered as the ambient air, the value R_{out} was calculated using the following formula [6]:

$$T_{env} = T_{gr}, \quad \frac{T_{avg} - T_{gr}}{R_{out}} = \frac{(T_{avg} - T_{gr})\bar{R}_0 - (\bar{T}_{avg} - T_{gr})R_0}{R_0\bar{R}_0 - R_{inf}^2} \quad (7)$$

$$R_{inf} = \frac{1}{2\pi\lambda_{gr}} \ln \sqrt{1 + \left(\frac{2H}{s}\right)^2}$$

where: T_{gr} is the soil temperature

T_{avg}, \bar{T}_{avg} are the average mean of temperatures respectively; considered and neighboring pipes;

R_{inf} is the additional thermal resistance of heat transfer between two pipes in the ground;

λ_{gr} is the thermal conductivity coefficient of the surrounding soil;

H is the depth of the pipeline to its axis;

s is the the distance between the axes of the pipe in question and the adjacent pipe;

R_0, \bar{R}_0 are the total thermal resistances of the pipe in question and the adjacent pipe without taking into account the mutual influence of their temperature fields.

The values R_0, \bar{R}_0 are calculated using formula, where corresponding R_{out}, \bar{R}_{out} values R_{out}, \bar{R}_{out} are equal to the thermal R_{gr} soil resistance R_{gr} , which is determined using Forchheimer formula [7]:

$$R_{gr} = \frac{1}{2\pi\lambda_{gr}} \ln \left(\frac{2H}{D_2} + \sqrt{\left(\frac{2H}{D_2}\right)^2 - 1} \right) \quad (8)$$

When laying a pipeline underground in a channel, the air in the channel was considered as the environment, the R_{out} value was calculated as the sum of the thermal resistances of successive heat transfers in the channel:

$$\begin{aligned} T_{env} &= T_{air}^{can}, & R_{out} &= R_{air}^{can} + R_{wall}^{can} + R_{gr}^{can} \\ R_{air}^{can} &= \frac{1}{\alpha_{air}^{can}\pi D_2}, & R_{wall}^{can} &= \frac{1}{\alpha_{air}^{can}\pi D_{can}}, & D_{can} &= \frac{2d_{can}h_{can}}{d_{can} + h_{can}} \end{aligned} \quad (9)$$

$$R_{gr}^{can} = \frac{\ln \left(3.5 \frac{Hh_{can}}{d_{can}^2} \right)}{\lambda_{gr} \left(5.7 + \frac{d_{can}}{2h_{can}} \right)}$$

where $R_{air}^{can}, R_{wall}^{can}, R_{gr}^{can}$ are the values of corresponding thermal resistances: heat exchange from the surface of the insulated pipeline into the air space of the channel, heat exchange from the air to the ground, heat exchange around the channel;

α_{air}^{can} is the heat exchange coefficient of air in the channel, according to [8] was accepted $8 \text{ W}/(\text{m}^2 \times ^\circ\text{C})$

$D_{can}, d_{can}, h_{can}$ is the equivalent diameter, width and height of channel
 T_{air}^{can} is the channel air temperature, which was calculated according to the next ratio for thermal resistances:

$$\frac{T_{air}^{can} - T_{avg}}{R_{ins} + R_{air}^{can}} + \frac{T_{air}^{can} - \bar{T}_{avg}}{\bar{R}_{ins} + \bar{R}_{air}^{can}} + \frac{T_{air}^{can} - T_{gr}}{R_{wall}^{can} + R_{gr}^{can}} = 0 \quad (10)$$

Where \bar{R}_{ins} , \bar{R}_{air}^{can} are the corresponding values of thermal resistances (insulation coating and heat transfer from the surface of pipe into the air space of the channel) [9].

As can be seen from the formulas (7)–(10) calculating temperature at the end of an underground pipe section requires calculating the water temperatures in adjacent pipe, which also depends on the temperature in the pipe in question. Therefore, during underground installation, an iterative process is used to calculate the final temperature of the section until the required accuracy is achieved [10].

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Conclusions

A formatted mathematical model of the relationship between heat supply system parameters was defined, serving as the basis for developing a flow calculation algorithm considering various system operation scenarios. Using the initial temperature T_0 at the inlet pipe of a section of the heating network and coolant flow rate q through this section, the corresponding final temperatures T_{end} were determined using formulas (9)-(11).

Temperature distribution across the entire heating network was defined using breadth-first search (graph traversal method), beginning from the nodes of heating network (thermal power plants, heating plants), for which temperatures in outlet were set. The heating network nodes were bypassed in the positive direction of flow ($q > 0$). When bypassing the heating network nodes, the temperature T_0 at the inlet of subsequent sections of the heating network (the temperature at the outlet of the initial node of the section) was calculated based on the thermal energy balance:

$$T_0 = \frac{\sum_{i=1}^n (q_i \cdot (T_{end})_i)}{\sum_{i=1}^n q_i} - \Delta T \quad (11)$$

where n is the number of nodes in heating network from which the flow comes to the current node;

$q_i, (T_{end})_i$ is the values of water flow rate and final temperature at the i -th section of the pipe entering the current node;

ΔT is the temperature loss in current node.

If the current node is a consumer in heat supply system, then $\Delta T > 0$, otherwise $\Delta T = 0$.

The software code for solving the problem of temperature distribution in a heating network based on heat exchange with the environment was written in C# and implemented as a separate software module within the System. Implementing a digital twin using the calculated model will enable real-time diagnostics of the heating system's condition, determining process parameters at any given time, and predicting parameter fluctuations and emergency situations.

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ЖЫЛУ ЖЕЛІСІНДЕГІ ТЕМПЕРАТУРАНЫ БӨЛҮДІҢ МАТЕМАТИКАЛЫҚ МОДЕЛІН ЖАСАУ

Бұл мақалада Павлодар жылу желілерінің мысалын қолдана отырып, жылу жүйесінің сандық егізін жасау үшін салқындатқыш ағынының таралуының математикалық моделі қарастырылады. Мұндай жүйелерді модельдеу температура, қысым, ағын жылдамдығы және құбыр сипаттамалары сияқты әртүрлі параметрлерді ескере отырып, салқындатқыштың таралуын оңтайландыруға мүмкіндік береді. Бұл мысал үшін қоршаған ортамен жылу алмасуына негізделген жылу желісіндегі температураның таралуының математикалық моделі жасалды және жер үсті және жер асты (құбыр) орнату нұсқалары қарастырылды. Желі түйіндері арасындағы құбырдың ұзындығы бойынша температураның таралуының сипаттамасы пайдаланылды.

Зерттеудің мақсаты – жылу жүйелеріндегі салқындатқыш ағындарын есептеу және оңтайландыру үшін пайдалануға болатын тиімді математикалық модельді жасау. Модельді жаңа жүйелерді жобалауға және қолданыстағы жүйелерді жаңғыртуға, сондай-ақ нақты уақыт режимінде салқындатқыш ағындарын жедел басқаруға қолдануға болады.

Зерттеудің негізгі мақсаттарына салқындатқыш ағынын модельдеудің қолданыстағы әдістері мен тәсілдерін талдау, жылу желілерінің нақты сипаттамаларын ескере отырып, математикалық модельді әзірлеу және эксперименттік деректерге негізделген модельді тексеру және валидациялау кіреді. Зерттеу нәтижелері жылумен жабдықтау мамандары, жобалау ұйымдары, пайдалану компаниялары және жылумен жабдықтау жүйелерінің энергия тиімділігі мен оңтайландыруы бойынша жұмыс істейтін зерттеушілер үшін пайдалы болуы мүмкін.

Кілтті сөздер: математикалық модель, сандық егіз, жылумен қамту, жылулық есептеу, гидравликалық есептеу.

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РАЗРАБОТКА МАТЕМАТИЧЕСКОЙ МОДЕЛИ РАСПРЕДЕЛЕНИЯ ТЕМПЕРАТУР В ТЕПЛОСЕТИ

В работе рассматривается математическая модель распределения потоков теплоносителя, для создания цифрового двойника системы теплоснабжения на примере тепловых сетей города Павлодар. Моделирование таких систем позволяет оптимизировать распределение теплоносителя с учётом различных параметров, таких как температура, давление, расход и характеристики трубопроводов. Для примера разработана математическая модель распределения температур в теплосети исходя из процесса теплообмена с окружающей средой, рассмотрены варианты надземной и подземной (канальной) прокладки. Использовано описание распределение температуры по длине трубопровода между узлами сети.

Целью исследования является создание эффективной математической модели, которая может быть использована для расчёта и оптимизации потоков теплоносителя в системах теплоснабжения. Модель может применяться для проектирования новых и модернизации существующих систем, а также для оперативного управления потоками теплоносителя в реальном времени.

Основные задачи исследования включают анализ существующих методов и подходов к моделированию потоков теплоносителя, разработку математической модели с учётом особенностей тепловых сетей, а также верификацию и валидацию модели на основе экспериментальных данных.

Результаты исследования могут быть полезны для специалистов в области теплоснабжения, проектных организаций, эксплуатирующих компаний и исследователей, занимающихся вопросами энергоэффективности и оптимизации систем теплоснабжения.

Ключевые слова: математическая модель, цифровой двойник, теплоснабжение, , тепловой расчет, гидравлический расчет.

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