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## **MATHEMATICAL MODELING OF NON-SINUSOIDAL MODES OF TRANSMISSION LINES**

*Mathematical modeling of non-sinusoidal modes in power transmission lines is key to understanding and evaluating their operation in real conditions. This paper addresses the need to develop a model that takes into account nonlinearities and higher harmonics, which are widely present in modern power transmission networks. Operational data and energy audit results indicate the presence of non-sinusoidal modes, which requires the development of adequate mathematical models for accurate analysis.*

*The study of multi-conductor transmission lines involves the use of mathematical models including telegraphic equations. However, the full adequacy of such a model requires additional consideration of several aspects, including two chain lines, surface effect, presence of lightning protection cables and ground effects on power transmission processes.*

*This paper presents a mathematical model that takes into account the above aspects for multi-conductor transmission lines. The features of two circuit lines, the surface effect in the wires, the presence of lightning protection cables and the influence of the earth on power transmission are taken into account.*

*This model is the result of improving and extending the existing mathematical framework for analyzing multi-conductor power transmission systems, which significantly improves the accuracy and*

*reliability of predictions and calculations in the field of electrical engineering and electric power engineering.*

*Keywords: telegraphic equations, power supply system, power losses, loss reduction, non-sinusoidal modes, Cayley-Hamilton theorem.*

### **Introduction.**

Non-sinusoidal modes in the operation of modern power lines are a given, which is confirmed not only by publications, but also by the materials of energy audits conducted to identify higher harmonic components [1]. For a full-fledged study of non-sinusoidal modes it is required not only to state the fact of its presence, but also to draw up a proper model of the power line taking into account the factors maximally approaching the results of calculations to the operational ones. It is important to take into account the requirements to such a model. The mathematical model of non-sinusoidal mode should take into account the following features:

- distributed line parameters for higher frequencies;
- surface effect in conductors for higher harmonics;
- the possibility of calculating the main and higher harmonics of voltages and currents.

The fulfillment of the above tasks is satisfied by a mathematical model in the form of telegraphic equations for multi-conductor transmission lines [2]. But this model requires additional consideration of:

- two circuit lines;
- surface effect;
- lightning protection cables;
- ground influence.

**Materials and Methods.** For multiconductor lines the mathematical model meeting the given requirements looks as follows

$$\begin{aligned}\dot{U}_x(\omega_n) &= Ae^{-\lambda_u(\omega_n)x} + Be^{\lambda_u(\omega_n)x}; \\ \dot{I}_x(\omega_n) &= Ce^{-\lambda_i(\omega_n)x} + De^{\lambda_i(\omega_n)x}.\end{aligned}\tag{1.1}$$

$$\begin{aligned}\frac{d\dot{U}_x}{dx} &= -\dot{Z}(\omega_n)\dot{I}_x(\omega_n) = -\lambda_u(\omega_n)e^{-\lambda_u(\omega_n)x}A(\omega_n) + \\ &\quad + \lambda_u(\omega_n)e^{\lambda_u(\omega_n)x}B(\omega_n); \\ \frac{d\dot{I}_x}{dx} &= -\dot{Y}(\omega_n)\dot{U}_x(\omega_n) = -\lambda_i(\omega_n)e^{-\lambda_i(\omega_n)x}C(\omega_n) + \lambda_i(\omega_n)e^{\lambda_i(\omega_n)x}D(\omega_n);\end{aligned}\tag{1.2}$$

Here  $\dot{U}_x(\omega_n), \dot{I}_x(\omega_n)$  – column vectors of complex voltages and currents of frequency  $\omega_n$  –  $n$ -th harmonic of dimension  $m \times 1$  in the section at a distance  $x$  from the beginning of the line;

$\lambda_u, \lambda_i$  – complex square matrices of dimension  $m \times m$ , representing corresponding functions from matrices of eigenvalue and mutual impedances and conductivities of wires and cables at the frequency of the  $n$ -th harmonic  $Z(\omega_n), Y(\omega_n)$  of dimension also  $m \times m$ :  $\lambda_u(\omega_n) = \sqrt{Z(\omega_n)Y(\omega_n)}$ ,  $\lambda_i(\omega_n) = \sqrt{Y(\omega_n)Z(\omega_n)}$ .

$A(\omega_n), B(\omega_n), C(\omega_n), D(\omega_n)$  are constant column vectors of dimension  $m \times 1$  calculated from the boundary conditions of 1.2 at the frequency of the  $n$ -th harmonic:

at the beginning of the line:

$$-\dot{Z}(\omega_n)\dot{I}_H(\omega_n) = -\lambda_u(\omega_n)A(\omega_n) + \lambda_u(\omega_n)B(\omega_n); \quad (1.3)$$

$$-\dot{Y}(\omega_n)\dot{U}_H(\omega_n) = -\lambda_i(\omega_n)C(\omega_n) + \lambda_i(\omega_n)D(\omega_n);$$

down the line:

$$-\dot{Z}(\omega_n)\dot{I}_K(\omega_n) = -\lambda_u(\omega_n)e^{-\lambda_u(\omega_n)l}A(\omega_n) + \lambda_u(\omega_n)e^{\lambda_u(\omega_n)l}B(\omega_n); \quad (1.4)$$

$$-\dot{Y}(\omega_n)\dot{U}_K(\omega_n) = -\lambda_i(\omega_n)e^{-\lambda_i(\omega_n)l}C(\omega_n) + \lambda_i(\omega_n)e^{\lambda_i(\omega_n)l}D(\omega_n).$$

The total number of mode parameters at the frequency of the  $n$ -th harmonic component is  $16m$  and includes real and imaginary components of voltages and currents of all wires and cables at the beginning and end of the line, as well as real and imaginary components of column vectors  $A(\omega_n), B(\omega_n), C(\omega_n), D(\omega_n)$ .

For convenience of transformation  $B$  the above equations, we can divide the parameters into mode dependent and mode independent. The process will consist of the following steps:

1) Determining the column vector  $A(\omega_n), B(\omega_n)$  by solving a system of matrix equations based on the first equation of the system (1.1) for the beginning and end of the line:

$$\dot{U}_H(\omega_n) = A(\omega_n) + B(\omega_n);$$

$$\dot{U}_K(\omega_n) = e^{-\lambda_u(\omega_n)l}A(\omega_n) + e^{\lambda_u(\omega_n)l}B(\omega_n).$$

Here  $l$  is the length of the line, and the indices  $n$  and  $k$  denote the beginning and end of the line.

The solution of the system when written in block form has the form:

$$\begin{aligned} \begin{vmatrix} A(\omega_n) \\ B(\omega_n) \end{vmatrix} &= \begin{vmatrix} E & E \\ e^{-\lambda_u(\omega_n)l} & e^{\lambda_u(\omega_n)l} \end{vmatrix}^{-1} \times \begin{vmatrix} \dot{U}_H(\omega_n) \\ \dot{U}_K(\omega_n) \end{vmatrix} = \\ &= \begin{vmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{vmatrix} \times \begin{vmatrix} \dot{U}_H(\omega_n) \\ \dot{U}_K(\omega_n) \end{vmatrix}, \end{aligned}$$

where  $E$  is a unit complex matrix of dimension  $m \times m$ .

whence

$$\begin{aligned} A(\omega_n) &= H_{11}\dot{U}_H(\omega) + H_{12}\dot{U}_K(\omega) \\ B(\omega_n) &= H_{21}\dot{U}_H(\omega) + H_{22}\dot{U}_K(\omega). \end{aligned}$$

2) Determination of the vector columns  $C(\omega_n)$ ,  $D(\omega_n)$  by solving the second equation from the system of matrix equations (1.1) for the beginning and end of the line:

$$\begin{aligned} \dot{I}_H(\omega_n) &= C + D \\ \dot{I}_K(\omega_n) &= Ce^{-\lambda_i(\omega_n)x} + De^{\lambda_i(\omega_n)x}. \end{aligned}$$

The solution of the system when using block matrix notation is of the form:

$$\begin{aligned} \begin{vmatrix} C(\omega_n) \\ D(\omega_n) \end{vmatrix} &= \begin{vmatrix} E & E \\ e^{-\lambda_u(\omega_n)l} & e^{\lambda_u(\omega_n)l} \end{vmatrix}^{-1} \times \begin{vmatrix} \dot{I}_H(\omega_n) \\ \dot{I}_K(\omega_n) \end{vmatrix} = \\ &= \begin{vmatrix} W_{11} & W_{12} \\ W_{21} & W_{22} \end{vmatrix} \times \begin{vmatrix} \dot{I}_H(\omega_n) \\ \dot{I}_K(\omega_n) \end{vmatrix}, \end{aligned}$$

whence

$$\begin{aligned} C(\omega_n) &= W_{11}\dot{I}_H(\omega) + W_{12}\dot{I}_K(\omega). \\ D(\omega_n) &= W_{21}\dot{I}_H(\omega) + W_{22}\dot{I}_K(\omega). \end{aligned}$$

3) Determination of the vector-column voltages and currents at the beginning and end of the line  $\dot{U}_H(\omega_n)$ ,  $\dot{U}_K(\omega_n)$ ,  $\dot{I}_H(\omega_n)$ ,  $\dot{I}_K(\omega_n)$  using the system equations (1.3, 1.4) taking into account certain values of the integration constant vectors  $A(\omega_n)$ ,  $B(\omega_n)$ ,  $C(\omega_n)$ ,  $D(\omega_n)$ :

$$\begin{aligned} Y(\omega_n)U_H(\omega_n) - \lambda_i(\omega_n)[W_{11}I_H(\omega_n) + W_{12}I_K(\omega_n)] + \\ + \lambda_i(\omega_n)[W_{21}I_H(\omega_n) + W_{22}I_K(\omega_n)] = 0 \end{aligned}$$

$$\begin{aligned} Y(\omega_n)U_K(\omega_n) - \lambda_i(\omega_n)e^{-\lambda_i(\omega_n)l}[W_{11}I_H(\omega_n) + W_{12}I_K(\omega_n)] + \\ + \lambda_i(\omega_n)e^{\lambda_i(\omega_n)l}[W_{21}I_H(\omega_n) + W_{22}I_K(\omega_n)] = 0 \end{aligned}$$

$$\begin{aligned} Z(\omega_n)I_K(\omega_n) - \lambda_u(\omega_n)[H_{11}U_H(\omega_n) + H_{12}U_K(\omega_n)] + \\ + \lambda_u(\omega_n)[H_{21}U_H(\omega_n) + H_{22}U_K(\omega_n)] = 0 \end{aligned}$$

$$Z(\omega_n)I_K(\omega_n) - \lambda_u(\omega_n)e^{-\lambda_u l}[H_{11}U_H(\omega_n) + H_{12}U_K(\omega_n)] + \\ + \lambda_u(\omega_n)e^{\lambda_u l}[H_{21}U_H(\omega_n) + H_{22}U_K(\omega_n)] = 0$$

The most difficult procedure in these calculations is to compute functions from matrices:  $\lambda_u(\omega_n) = \sqrt{Z(\omega_n)Y(\omega_n)}$ ,  $\lambda_i(\omega_n) = \sqrt{Y(\omega_n)Z(\omega_n)}$ ,  $e^{-\lambda_u(\omega_n)l}$ ,  $e^{\lambda_u(\omega_n)l}$ ,  $e^{-\lambda_i(\omega_n)l}$ ,  $e^{\lambda_i(\omega_n)l}$ . The computation of these functions is performed using the Cayley-Hamilton theorem [3], where the matrix function is defined as

$$f(A) = \frac{1}{\Delta} \sum_{k=1}^n \Delta_{n-k} A^{n-k},$$

here  $\Delta$  is the Vandermonde determinant of  $\det[\lambda_i^{k-1}]$ ,

$$W(\lambda_1, \lambda_2, \dots, \lambda_n) = \begin{pmatrix} 1 & \lambda_1 & \lambda_1^2 & \dots & \lambda_1^{n-1} \\ 1 & \lambda_2 & \lambda_2^2 & \dots & \lambda_2^{n-1} \\ 1 & \lambda_3 & \lambda_3^2 & \dots & \lambda_3^{n-1} \\ \dots & \dots & \dots & \dots & \dots \\ 1 & \lambda_n & \lambda_n^2 & \dots & \lambda_n^{n-1} \end{pmatrix}$$

where  $\lambda_i$  are the eigenvalues of the matrix  $A$ ;

$\Delta_j$  is the determinant obtained if in  $\Delta$  instead of  $\lambda_1^j, \lambda_2^j, \dots, \lambda_n^j$  we substitute the values of the functions  $f(\lambda_1), f(\lambda_2), \dots, f(\lambda_n)$ .

**Results and discussion.** In those cases when the harmonic components of the load currents are not known in advance and are determined during the calculation process depending on the measured voltage spectra using appropriate mathematical models, e.g. [4; 5] the calculation should be performed iteratively.

Evaluation of the obtained results using a new mathematical model for multi-conductor transmission lines. This involves comparing the model data with experimental measurements or with data obtained using other models [1].

A discussion of how effectively accounting for two chain lines, surface effects, lightning protection cables and ground effects in the model improves the accuracy and fit of the model to real power transmission conditions [6; 7].

Consideration of the effect of each of the considered aspects on the power transmission characteristics of a multi-chain line system. For example, how the transmission parameters change when the surface effect or lightning protection ropes are taken into account.

A discussion of the limitations of the model and possible ways to improve or extend it. This may include pointing out missing aspects that have not been accounted for in this model and opportunities for further research in this area [8; 9].

A discussion of how the results obtained can be useful for practical applications in the electric power industry, transmission network design, or for power system optimization [10].

These aspects of the results and discussions can serve as a basis for understanding the applicability and significance of the new mathematical model for multi-conductor transmission lines in engineering and scientific practice.

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#### **Conclusion.**

The presented equations allow the study of the modes of lines with different number of wires and lightning protection cables. Also on the basis of these equations it is possible to determine power losses, and at measurements on daily and more intervals calculation of electric energy losses

Non-sinusoidal modes in transmission lines require a suitable mathematical model to account for distributed parameters, surface effects and higher harmonics, which will provide more accurate research results.

It is important to consider distributed line parameters for higher frequencies, surface effects in conductors and the ability to calculate both fundamental and higher harmonics of voltages and currents.

In cases where the harmonics of load currents are not known in advance and are determined during the calculation process, iterative methods are used for accurate modeling taking into account the measured voltage spectra.

The developed equations allow studying different modes of transmission lines with different number of wires and lightning protection ropes, as well as determining power and electrical energy losses at different time intervals.

This work highlights the importance of mathematical modeling for more accurate analysis of non-sinusoidal modes in transmission lines, which is of key importance for efficient and reliable operation of power grids.

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## МАТЕМАТИЧЕСКОЕ МОДЕЛИРОВАНИЕ НЕСИНУСОИДАЛЬНЫХ РЕЖИМОВ ЛИНИЙ ЭЛЕКТРОПЕРЕДАЧИ

*Математическое моделирование несинусоидальных режимов в линиях электропередачи является ключевым для понимания и оценки их работы в реальных условиях. В данной работе рассматривается необходимость разработки модели, учитывающей нелинейности и высшие гармоники, которые широко присутствуют в современных сетях передачи электроэнергии. Эксплуатационные данные и результаты энергоаудита свидетельствуют о наличии несинусоидальных режимов, что требует создания адекватных математических моделей для точного анализа.*

*Исследование многопроводных линий электропередачи подразумевает использование математических моделей, включающих телеграфные уравнения. Однако, для полной адекватности такой модели требуется дополнительный учет нескольких аспектов, включая две цепные линии, поверхностный эффект, наличие грозозащитных тросов и воздействие земли на процессы передачи электроэнергии.*

*В данной статье представлена математическая модель, учитывающая перечисленные аспекты для многопроводных линий электропередачи. Учтены особенности двух цепных линий, поверхностный эффект в проводниках, наличие грозозащитных тросов и влияние земли на передачу электроэнергии.*

*Эта модель является результатом усовершенствования и расширения существующей математической базы для анализа многопроводных систем передачи электроэнергии, что существенно повышает точность и достоверность прогнозов и расчетов в сфере электротехники и электроэнергетики.*

*Ключевые слова: телеграфные уравнения, система электроснабжения, потери электрической энергии, снижение потерь, несинусоидальные режимы, теорема Кэли-Гамильтона.*

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## ЭЛЕКТР ЖЕЛІЛЕРІНІҢ СИНУСОИДАЛЫ ЕМЕС РЕЖИМДЕРІН МАТЕМАТИКАЛЫҚ МОДЕЛЬДЕУ

*Электр желілеріндегі синусоидалы емес режимдерді математикалық модельдеу олардың нақты әлемдегі жұмысын түсіну және бағалау үшін маңызды. Бұл жұмыста қазіргі заманғы электр беру желілерінде кеңінен кездесетін сызықтық емес және жоғары гармониканы ескеретін модель жасау қажеттілігі қарастырылады. Пайдалану деректері мен энергия аудитінің нәтижелері синусоидалы емес режимдердің болуын көрсетеді, бұл дәл талдау үшін барабар математикалық модельдерді құруды талап етеді.*

*Көп сымды электр желілерін зерттеу телеграф теңдеулерін қамтитын математикалық модельдерді қолдануды қамтиды. Алайда, мұндай модельдің толық сәйкестігі бірнеше аспектілерді, соның ішінде екі тізбекті желіні, беттік эффектін, найзағайдан қорғайтын кабельдердің болуын және жердің электр энергиясын беру процестеріне әсерін қосымша есепке алуды қажет етеді.*

*Бұл мақалада көп сымды электр желілері үшін аталған аспектілерді ескеретін математикалық модель берілген. Екі тізбекті желінің ерекшеліктері, сымдардағы беттік әсер,*

*найзагайдан қорғайтын кабельдердің болуы және жердің электр энергиясын беруге әсері ескеріледі.*

*Бұл модель электр энергетикасы мен электр энергетикасы саласындағы болжамдар мен есептеулердің дәлдігі мен сенімділігін едәуір арттыратын электр энергиясын берудің көп сымды жүйелерін талдау үшін қолданыстағы математикалық базаны жетілдіру мен кеңейтудің нәтижесі болып табылады.*

*Кілтті сөздер: телеграф теңдеулері, электрмен жабдықтау жүйесі, электр энергиясының жоғалуы, шығынның төмендеуі, синусоидалы емес режимдер, Кейли-Гамильтон теоремасы.*

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