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SYNTHESIS OF SELF-ORGANIZING CONTROL FOR MIMO SYSTEMS VIA STRUCTURALLY STABLE MAPPINGS

The aim of this paper is to develop a synthesis method for a self-organizing control system for linear objects with m-inputs and n-outputs within the class of two-parameter structurally stable mappings. The proposed approach is based on the gradient-velocity method of Lyapunov vector functions. Conditions for aperiodic robust stability of stationary states are derived in the form of systems of parametric inequalities that define admissible regions of parameter variations. It is shown that transitions between stable states may occur due to bifurcations, which are interpreted as a manifestation of self-organization. A procedure for selecting a self-organizing model with prescribed transient characteristics is developed, along with a method for computing controller coefficients based on matching the stability conditions of the model and the control system. The scientific novelty of the work lies in the development of a unified synthesis framework that ensures aperiodic transient responses, robustness under parameter uncertainty, and suppression of oscillatory and chaotic regimes.

Keywords: self-organization, MIMO systems, bifurcation, Lyapunov method, robust stability, structurally stable mappings.

Introduction

The design of control systems capable of operating under uncertainty remains one of the central problems of modern control theory. In practical applications, systems are often affected by parameter variations, external disturbances, and modeling inaccuracies, which may significantly degrade performance and

lead to instability [1; 2]. These challenges become particularly critical for multidimensional systems with multiple inputs and outputs, where interactions between channels increase the complexity of the dynamic behavior and complicate the synthesis of control laws [3; 4].

Recent developments in nonlinear dynamics have demonstrated that even systems with relatively simple structures may exhibit multiple equilibrium states and undergo qualitative changes in behavior as parameters vary [5; 6]. Such phenomena are typically associated with bifurcations and may result in transitions between different dynamic regimes. While these effects introduce additional difficulties in control design, they also provide a basis for constructing adaptive control strategies that exploit the structural properties of the system.

One of the promising approaches in this direction is the use of structurally stable mappings derived from catastrophe theory [7; 8]. These mappings make it possible to represent the system dynamics in a form that explicitly accounts for the existence and transformation of equilibrium states. As a result, it becomes possible to influence system behavior through controlled variation of parameters, which is particularly important in the context of self-organizing systems.

Self-organizing control systems are characterized by the ability to modify their parameters or structure during operation in response to changes in the environment [9]. This property allows them to maintain stable operation even in the presence of uncertainty. A key requirement for such systems is the preservation of stability over a wide range of parameter variations, rather than only in a local neighborhood of equilibrium.

In this context, the concept of aperiodic robust stability plays an important role. It ensures that system trajectories converge monotonically to equilibrium states without oscillations or chaotic behavior, which is essential for reliable operation in engineering applications. The analysis of such stability properties requires the development of analytical methods capable of describing system behavior in multidimensional spaces.

An effective tool for this purpose is the gradient-based approach using Lyapunov vector functions, which allows the stability conditions to be formulated in terms of parameter constraints [10; 11]. This framework provides a systematic way to determine admissible regions of system parameters ensuring stable and non-oscillatory dynamics [12].

Despite the existing studies in this field, most of the available results are limited to systems of relatively low dimensionality. In particular, approaches developed for single-input single-output systems cannot be directly extended to the general multidimensional case without additional considerations [13; 14]. Therefore, the problem of synthesizing self-organizing control systems for objects with multiple inputs and outputs remains insufficiently explored.

The objective of this paper is to develop a synthesis method for self-organizing control systems for linear objects with m-inputs and n-outputs within the class of two-parameter structurally stable mappings. The proposed approach is aimed at ensuring aperiodic robust stability and achieving desired transient characteristics. The solution of this problem includes deriving stability conditions for stationary states, analyzing their changes under parameter variations, selecting a self-organizing model with required properties, and determining controller parameters based on consistency between the model and the control system.

Materials and methods

A synthesis framework for self-organizing control of linear MIMO systems is investigated using two-parameter structurally stable mappings as the underlying representation.

Consider a linear dynamic system described by

$$\dot{x} = Ax(t) + Bu(t) \tag{1}$$

Where $x \in R^n$ is the state vector, and A and B are system matrices with uncertain parameters.

$$A = \begin{Vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{Vmatrix}, \quad B = \begin{Vmatrix} b_{11} & 0 & \dots & 0 \\ 0 & b_{22} & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & b_{nn} \end{Vmatrix}$$

The control input is formulated using a representation based on two-parameter structurally stable mappings and can be written as

$$u_i(t) = -x_i^4(t) - k_i^1 x_i^2(t) + k_i x_i(t), i = 1,\dots,n \tag{2}$$

Substituting (2) into (1), the closed-loop system can be written as

$$\begin{cases} \dot{x}_1 = -b_{11}x_1^4 - b_{11}k_1^1 x_1^2 + (a_{11} + b_{11}k_1)x_1 + a_{12}x_2 + a_{13}x_3 + \dots + a_{1n}x_n \\ \dot{x}_2(t) = a_{21}x_2 - b_{22}x_2^4 - b_{22}k_2^1 x_2^2 + (a_{11} + b_{22}k_2)x_2 + a_{23}x_3 + \dots + a_{2n}x_n \\ \dots \\ \dot{x}_n = a_{n1}x_1 + a_{n2}x_2 + \dots - b_{nn}x_n^4 + b_{nn}k_n^1 x_n^2 + (a_{nn} + b_{nn}k_n)x_n \end{cases} \tag{3}$$

The stationary states of the system are determined from the condition

$$\begin{cases} -b_{11}x_1^4 + b_{11}k_1^1 x_1^2 + (a_{11} + b_{11}k_1)x_1 + a_{12}x_2 + a_{13}x_3 + \dots + a_{1n}x_n = 0 \\ a_{21}x_1 - b_{22}x_2^4 - b_{22}k_2^1 x_2^2 + (a_{22} + b_{22}k_2)x_2 + a_{23}x_3 + \dots + a_{2n}x_n = 0 \\ \dots \\ a_{n1}x_1 + a_{n2}x_2 + \dots - b_{nn}x_n^4 - b_{nn}k_n^1 x_n^2 + (a_{nn} + b_{nn}k_n)x_n = 0 \end{cases} \tag{4}$$

It follows from (4) that the trivial stationary state can be written as

$$x_{is}^{1} = 0, i = 1,\ldots,n \tag{5}$$

Apart from the trivial solution, the system also possesses additional equilibrium states, which are determined by

$$-b_{ii}x_{is}^{3} - b_{ii}k_i^{1}x_{is}^{2} + (a_{ii} + b_{ii}k_i) = 0, i = 1,\ldots,n \tag{6}$$

Equation (6) leads to the following solutions

$$\begin{cases} x_{is}^{2} = \sqrt[3]{\dfrac{b_{ii}k_i - a_{ii}}{2b_{ii}}}, i = 1,\ldots,n \\ x_{is}^{3,4} = -\sqrt[3]{\dfrac{b_{ii}k_i - a_{ii}}{2b_{ii}}}, k_i^{1} = \sqrt[3]{\left(\dfrac{b_{ii}k_i - a_{ii}}{2b_{ii}}\right)^{2}} \end{cases} \tag{7}$$

The aperiodic robust stability of the stationary state (5) is examined using a gradient-based approach involving Lyapunov vector functions. The analysis is carried out for the system defined in (3), which is constructed using two-parameter structurally stable mappings.

The gradient-velocity approach relies on a geometric view of stability analysis associated with Lyapunov's second method and incorporates ideas from gradient dynamical systems. In this framework, the system dynamics can be represented in the form

$$\left(\frac{dx_i}{dt}\right)_{x_j} = -\frac{\partial V_i(x)}{\partial x_j}, i = 1,\ldots,n; j = 1,\ldots,n.$$

Where $x(t) \in R^n$ is the state vector of the dynamic system, and $V_i(x)$ is the Lyapunov vector function, whose components can be interpreted as potential functions.

Within this formulation, the system dynamics are governed by the gradient components of the Lyapunov vector function, while the velocity vector is expressed in terms of the state-space coordinates [15].

The conditions ensuring aperiodic robust stability follow from the requirement that the Lyapunov vector function *V(x)* is positive definite. According to the Morse lemma [16], it can be locally approximated by a quadratic form. As a result, the stability conditions can be formulated as a system of parameter inequalities consistent with classical robust control theory [17; 18]:

$$\begin{cases} -(a_{11} + b_{11}k_1 + a_{21} + a_{31} + \ldots + a_{n1}) > 0 \\ -(a_{12} + a_{22} + b_{22}k_2 + a_{32} + \ldots + a_{n2}) > 0 \\ -(a_{13} + a_{23} + a_{33} + b_{33}k_3 + \ldots + a_{n3}) > 0 \\ \qquad\qquad \cdots \\ -(a_{1n} + a_{2n} + a_{3n} + \ldots + a_{nn} + b_{nn}k_n) > 0 \end{cases} \tag{8}$$

These inequalities (8) define the admissible parameter region in which the stationary state (5) is aperiodically robustly stable.

Depending on the system parameters, multiple stationary states may exist, which is a characteristic feature of systems constructed in the class of structurally stable mappings.

The evolution of the system is analyzed through the behavior of the time derivative of the Lyapunov function, in particular its sign. In this case, the derived inequalities guarantee monotonic convergence of system trajectories and exclude oscillatory and chaotic regimes.

To evaluate the aperiodic stability properties of the states (7), a gradient-based Lyapunov vector function method is applied. The system (3) is therefore reformulated in deviation variables with respect to the equilibrium (7).

$$\begin{cases} \dot{x}_1 = -b_{11}x_1^4 - 4b_{11}\sqrt[3]{\dfrac{b_{11}k_1 - a_{11}}{2b_{11}}}x_1^3 - 3b_{11}\sqrt[3]{\left(\dfrac{b_{11}k_1 - a_{11}}{2b_{11}}\right)^2}x_1^2 - \\ -(b_{11}k_1 - a_{11})x_1 - a_{12}x_2 - a_{13}x_3 -,\ldots,-a_{1n}x_n \\ \dot{x}_2 = -a_{22}x_1 - b_{22}x_2^4 - 4b_{22}\sqrt[3]{\dfrac{b_{22}k_2 - a_{22}}{2b_{22}}}x_2^3 - 3b_{22}\sqrt[3]{\left(\dfrac{b_{22}k_2 - a_{22}}{2b_{22}}\right)}x_2^2 - \\ -(b_{22}k_2 - a_{22})x_2 - a_{23}x_3 -,\ldots,-a_{2n}x_n \\ \qquad\qquad \ldots \\ \dot{x}_n = -a_{n1}x_1 - a_{n2}x_2 - a_{n3}x_3 - b_{nn}x_n^4 - 4b_{nn}\sqrt[3]{\dfrac{b_{nn}k_n^2 - a_{nn}}{2b_{nn}}}x_n^3 - \\ -3b_{nn}\sqrt[3]{\left(\dfrac{b_{nn}k_n - a_{nn}}{2b_{nn}}\right)^2}x_n^2 - \left(b_{nn}k_n - a_{nn}\right)x_n \end{cases} \tag{9}$$

Aperiodic stability properties of system (9) are investigated by applying a gradient-based Lyapunov vector function method [19].

The requirement that the Lyapunov vector function is positive definite, which is equivalent to a quadratic form, leads to the conditions ensuring aperiodic robust stability of the stationary state (7). These conditions can be written as the following system of inequalities:

$$\begin{cases} b_{11} > 0, b_{22} > 0, \ldots, b_{nn} > 0 \\ b_{11}k_1 - a_{11} - a_{21} - a_{31} - \ldots - a_{n1} \geq 0 \\ -a_{12} + b_{22}k_2 - a_{22} - a_{32} - \ldots - a_{n2} \geq 0 \\ -a_{13} - a_{23} + b_{33}k_3 - a_{33} - \ldots - a_{n3} \geq 0 \\ \qquad\qquad \ldots \\ -a_{1n} - a_{2n} - a_{3n} - \ldots + b_{nn}k_n - a_{nn} \geq 0 \end{cases} \tag{10}$$

The set of inequalities (10) characterizes the parameter domain in which the stationary state (7) remains aperiodically stable.

The stationary state (5) exists and preserves aperiodic stability under variations of uncertain parameters of system (3) within the admissible region defined by (8). When these conditions are violated, a bifurcation occurs, leading to the appearance of the stationary state (7). The latter remains stable in the aperiodic sense within the parameter domain specified by (10).

As a result, a self-organizing model is constructed in the class of two-parameter structurally stable mappings, ensuring the desired transient response and specified parameters. The problem addressed here involves the selection and analysis of a self-organizing control system within the class of two-parameter structurally stable mappings.

Assume a self-organizing model with prescribed transient characteristics obtained through simulation. The model is described by

$$\begin{cases} \dot{x}_1 = -x_1^4 - d_1^1 x_1^2 + d_1 x_1 + a_{12}x_2 + \ldots + a_{1n}x_n \\ \dot{x}_2 = a_{21}x_1 - x_2^4 - d_2^1 x_2^2 + d_2 x_2 + \ldots + a_{2n}x_n \\ \qquad\qquad \ldots \\ \dot{x}_n = a_{n1}x_1 + a_{n2}x_2 + \ldots - x_n^4 - d_n^1 x_n^2 + d_n x_n \end{cases} \tag{11}$$

The system (11) is analyzed for given values of parameters d_i^1 and d_i, $(i = 1, 2, ..., n)$ within a gradient-based framework employing Lyapunov vector functions. It is shown that the model (11) possesses self-organizing properties.

The stationary states of model (11) are determined from

$$\begin{cases} -x_1^4 - d_1^1 x_1^2 + d_1 x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = 0 \\ a_{21}x_1 - x_2^4 - d_2^1 x_2^2 + d_2 x_2 + \ldots + a_{2n}x_n \\ \qquad\qquad \ldots \\ a_{n1}x_1 + a_{n2}x_2 + \ldots - x_n^4 - d_n^1 x_n^2 + d_n x_n = 0 \end{cases} \tag{12}$$

From (12), the trivial stationary state is obtained as

$$x_{is}^1 = 0, i = 1, ..., n \tag{13}$$

Nontrivial stationary states are determined by

$$-x_{is}^3 + d_i^1 x_{is}^2 + d_i x_{is} = 0, i = 1,...,n. \tag{14}$$

The solutions of (14) are given by

$$\begin{cases} x_{is}^2 = \sqrt[3]{\dfrac{d_i}{2}}, i = 1,...,n \\ d_i^1 = \sqrt[3]{\left(\dfrac{d_i}{2}\right)^2}, x_{is}^{3,4} = -\sqrt[3]{\dfrac{d_i}{2}}, \quad i = 1,...,n \end{cases} \tag{15}$$

The stability properties of the stationary states (13) and (15) are examined within a gradient-based framework employing Lyapunov vector functions. System (11) is considered as a gradient system, for which the following relation holds: $\frac{dx_i}{dt} = -\frac{\partial V_i(x)}{\partial x}, i = 1,...,n$, where $V(x) = \left(V_1(x_1,...,x_n),...,V_n(x_1,...,x_n)\right)$ denotes the Lyapunov vector function.

The stability of the stationary state (13) of model (11) is evaluated by enforcing positive definiteness of the Lyapunov vector function *V(x)*. This leads to a set of inequalities that define the conditions for aperiodic robust stability:

$$\begin{cases} -(d_1 + a_{21} + a_{31} + ... + a_{n1}) > 0 \\ -(a_{12} + d_2 + a_{23} + ... + a_{n2}) > 0 \\ -(a_{13} + a_{23} + d_3 + ... + a_{n3}) > 0 \\ \qquad\qquad ... \\ -(a_{1n} + a_{2n} + a_{3n} + ... + d_n) > 0 \end{cases} \tag{16}$$

Thus, inequalities (16) define the parameter region in which the stationary state (13) remains aperiodically stable.

The stability of the stationary state (15) is evaluated using the same approach. For this purpose, system (11) is rewritten in deviation variables relative to the stationary state (15), resulting in the transformed system:

$$\begin{cases} \dot{x}_1 = -x_1^4 - 4\sqrt[3]{\dfrac{d_1}{2}}x_1^3 - 3\sqrt[3]{\left(\dfrac{d_1}{2}\right)^2}x_1^2 - d_1x_1 + a_{12}x_2 + a_{13}x_3 + \ldots + a_{1n}x_n \\ \dot{x}_2 = -x_2^4 - 4\sqrt[3]{\dfrac{d_2}{2}}x_2^3 - 3\sqrt[3]{\left(\dfrac{d_2}{2}\right)^2}x_2^2 - d_2x_2 + a_{21}x_1 + a_{23}x_3 + \ldots + a_{2n}x_n \\ \qquad\qquad ... \\ \dot{x}_n = a_{n1}x_1 + a_{n2}x_2 + ... - x_n^4 - 4\sqrt[3]{\dfrac{d_n}{2}}x_n^3 - 3\sqrt[3]{\left(\dfrac{d_n}{2}\right)^2}x_n^2 - d_nx_n \end{cases} \tag{17}$$

The requirement of positive definiteness of the Lyapunov vector function *V(x)* leads to the conditions ensuring aperiodic robust stability of the model in deviation form (17). These conditions can be expressed as

$$\begin{cases} d_1 - a_{21} - a_{31} - \ldots - a_{n1} > 0 \\ d_2 - a_{22} - a_{32} - \ldots - a_{n2} > 0 \\ d_3 - a_{13} - a_{23} - \ldots - a_{n3} > 0 \\ \qquad\qquad \ldots \\ d_n - a_{1n} - a_{2n} - \ldots - a_{n-1,n} > 0 \end{cases} \tag{18}$$

Thus, inequalities (18) define the parameter region in which the stationary state (15) remains aperiodically stable.

Model (11) remains stable over a wide range of parameter variations $d_i, i = 1,\ldots,n$, which confirms its self-organizing property. It follows that under conditions (16), the stationary state (13) exists and is stable, while under conditions (18), the stationary state (15) becomes aperiodically robustly stable. The transition between these states occurs through a bifurcation mechanism.

Simultaneous stability of states (13) and (15) is not observed, which follows from the structural characteristics of two-parameter mappings. This feature provides a basis for constructing self-organizing models that exclude unstable regimes and eliminate chaotic dynamics.

The determination of controller coefficients for a self-organizing control system is addressed for linear objects with multiple inputs and outputs within the framework of two-parameter structurally stable mappings.

The proposed approach relies on ensuring consistency between the aperiodic robust stability conditions of the control system and those of the self-organizing model with prescribed transient characteristics. In this framework, the desired performance is determined from simulation of the self-organizing model.

To ensure the transfer of dynamic properties from the model to the control system, the stability conditions of the control system (8), (10) are equated to the stability conditions of the model (16), (18). This yields the following system of equations obtained by equating (8) and (16):

$$\begin{cases} -(b_{11}k_1 + a_{11} + a_{21} + a_{31} + ,\ldots, + a_{n1}) = -(d_1 + a_{21} + a_{31} + ,\ldots, + a_{n1}) \\ -(a_{12} + b_{22}k_2 + a_{22} + a_{32} + ,\ldots, + a_{n2}) = -(a_{12} + d_2 + a_{32} + ,\ldots, + a_{n2}) \\ -(a_{13} + a_{23} + b_{33}k_3 + a_{33} + ,\ldots, + a_{n3}) = -(a_{13} + a_{23} + d_3 + ,\ldots, + a_{n3}) \\ \qquad\qquad \ldots \\ -(a_{1n} + a_{2n} + a_{3n} + ,\ldots, + b_{nn}k_n + a_{nn} = -(a_{1n} + a_{2n} + a_{3n} + ,\ldots, + d_n) \end{cases} \tag{19}$$

Similarly, by equating (10) and (18), the controller parameters are obtained as:

$$\begin{cases} b_{11}k_1 - a_{11} - a_{21} - a_{31} -,\dots,-a_{n1} = d_1 - a_{21} - a_{31} -,\dots,-a_{n1} \\ -a_{12} + b_{22}k_2 - a_{22} - a_{32} -,\dots,-a_{n2} = -a_{12} + d_2 - a_{32} -,\dots,-a_{n2} \\ -a_{13} - a_{23} + b_{33}k_3 - a_{33} -,\dots,-a_{n3} = -a_{13} - a_{23} + d_3 -,\dots,-a_{n3} \\ \dots \\ -a_{1n} - a_{2n} - a_{3n} -,\dots,+b_{nn}k_n - a_{nn} = -a_{1n} - a_{2n} - a_{3n} -,\dots,+d_n \end{cases} \qquad (20)$$

By matching the corresponding terms of systems (19) and (20), the controller coefficients for the self-organizing MIMO control system within the class of two-parameter structurally stable mappings can be determined as

$$k_1 = \frac{d_1 + a_{11}}{b_{11}}, k_2 = \frac{d_2 + a_{22}}{b_{22}}, k_3 = \frac{d_3 + a_{33}}{b_{33}},\dots, k_n = \frac{d_n + a_{nn}}{b_{nn}} \qquad (21)$$

Thus, the synthesis problem admits a unique solution. The resulting self-organizing control system ensures the desired control performance while preserving aperiodic transient behavior.

At the same time, the proposed approach eliminates the occurrence of deterministic chaotic regimes and prevents the formation of undesirable dynamic behaviors, such as strange attractors, by restricting the system operation to the admissible stability regions.

Results and Discussion

The results of the study show that the closed-loop system (3) possesses regions of aperiodic robust stability defined by the system of inequalities (8). Within these regions, the stationary state (5) remains stable under variations of uncertain parameters, and the transient responses exhibit a non-oscillatory (aperiodic) behavior.

Violation of conditions (8) leads to a loss of stability of the stationary state (5). As a consequence, new stationary states (7) arise, as defined by equation (6). When conditions (10) are satisfied, these states remain aperiodically stable.

Thus, variation of system parameters causes a bifurcation transition between stationary states, which reflects the self-organization process. In this case, the system automatically transitions to a new stable regime without the occurrence of oscillatory or chaotic behavior.

The analysis of the self-organizing model (11) shows that its stationary states (13) and (15) are determined by equations (12) and (14), while the corresponding stability regions are defined by conditions (16) and (18). The comparison of stability conditions for the model and the control system makes it possible to formulate a controller synthesis procedure.

Equating the stability conditions (8) and (16), as well as (10) and (18), leads to the systems of equations (19) and (20), the solution of which yields expression (21) for the controller parameters. This ensures the transfer of the desired transient characteristics from the model to the control system.

The obtained results confirm that the proposed method provides the formation of regions of aperiodic robust stability, the existence of multiple stationary states, and a controlled transition between them under parameter variations.

Conclusion

The paper introduces a method for constructing self-organizing control systems for multidimensional linear objects based on two-parameter structurally stable mappings. The analysis is carried out within a Lyapunov vector function framework, allowing the formulation of stability criteria in terms of parameter-dependent constraints. These constraints define regions in which the system maintains non-oscillatory and robust behavior despite parameter variations.

It has been shown that the proposed approach ensures the formation of aperiodic transient responses with monotonic convergence to equilibrium states. In addition, the method describes the mechanism of transitions between stable states through bifurcations, which reflects the self-organizing properties of the system.

The use of two-parameter structurally stable mappings and catastrophe theory makes it possible to eliminate the development of unstable and chaotic regimes, including deterministic chaos.

A procedure for matching the stability conditions of the model and the control system has been developed, allowing the controller parameters to be uniquely determined and ensuring the desired performance characteristics.

The obtained results extend the capabilities of robust control system synthesis and can be applied to the design of complex technical systems operating under uncertainty.

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МІМО ЖҮЙЕЛЕРІ ҮШІН ӨЗІН-ӨЗІ ҰЙЫМДАСТЫРАТЫН БАСҚАРУДЫ ҚҰРЫЛЫМДЫҚ ТҰРАҚТЫ БЕЙНЕЛЕУЛЕР НЕГІЗІНДЕ СИНТЕЗДЕУ

Бұл жұмыстың мақсаты – т-кірісі және п-шығысы бар сызықтық объектілер үшін екі параметрлі құрылымдық тұрақты бейнелеулер класында өзін-өзі ұйымдастыратын басқару жүйесін синтездеу әдісін әзірлеу болып табылады. Ұсынылған тәсіл Ляпуновтың вектор-функцияларының градиентті-жылдамдық әдісіне негізделген. Стационарлық күйлердің апериодтық робастты орнықтылығының шарттары параметрлердің өзгеруінің рұқсат етілген аймақтарын анықтайтын параметрлік теңсіздіктер жүйесі түрінде алынған. Жүйе параметрлеріне байланысты орнықты күйлер арасындағы ауысулар бифуркациялар нәтижесінде жүзеге асатыны және бұл өзін-өзі ұйымдастырудың көрінісі ретінде қарастырылатыны көрсетілген. Берілген өтпелі сипаттамалары бар өзін-өзі ұйымдастыратын модельді таңдау процедурасы, сондай-ақ модель мен басқару жүйесінің орнықтылық шарттарын сәйкестендіру негізінде реттегіш коэффициенттерін есептеу әдісі ұсынылған. Жұмыстың ғылыми жаңалығы апериодтық өтпелі процестерді, параметрлік анықталмағандық жағдайындағы робасттылықты және тербелмелі әрі хаостық режимдерді басуды қамтамасыз ететін біртұтас синтез әдісін әзірлеуде болып табылады.

Кілтті сөздер: өзін-өзі ұйымдастыру, МІМО жүйелері, бифуркация, Ляпунов әдісі, робастты тұрақтылық, құрылымдық тұрақты бейнелеулер.

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СИНТЕЗ САМООРГАНИЗУЮЩЕГО УПРАВЛЕНИЯ MIMO-СИСТЕМ НА ОСНОВЕ СТРУКТУРНО УСТОЙЧИВЫХ ОТОБРАЖЕНИЙ

Целью данной работы является разработка метода синтеза самоорганизующейся системы управления для линейных объектов с m-входами и n-выходами в классе двухпараметрических структурно устойчивых отображений. Предложенный подход основан на градиентно-скоростном методе вектор-функций Ляпунова. Получены условия апериодической робастной устойчивости стационарных состояний в виде систем параметрических неравенств, определяющих допустимые области изменения параметров. Показано, что переходы между устойчивыми состояниями могут происходить в результате бифуркаций, что интерпретируется как проявление самоорганизации. Разработана процедура выбора самоорганизующейся модели с заданными переходными характеристиками, а также метод вычисления коэффициентов регулятора на основе согласования условий устойчивости модели и системы управления. Научная новизна работы заключается в разработке единого подхода к синтезу, обеспечивающего апериодический характер переходных процессов, робастность при параметрической неопределённости и подавление колебательных и хаотических режимов.

Ключевые слова: самоорганизация; MIMO системы; бифуркация; метод Ляпунова; робастная устойчивость; структурно устойчивые отображения.

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